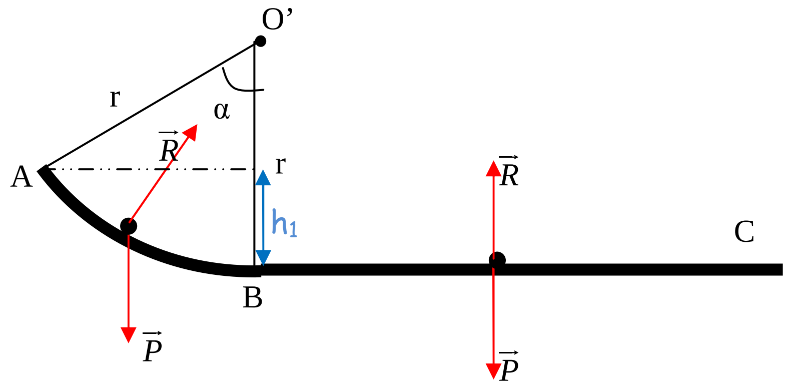
حل التـمـريـن 15

1. نطبق مبرهنة الطاقة الحركية بين A و B :



$$E_{CB}-E_{CA}=\sum W_{A\to B}(\vec{F})\Rightarrow \frac{1}{2}mv_B^2-\frac{1}{2}mv_A^2=W_{A\to B}(\vec{P})+W_{A\to B}(\vec{R})$$

$$v_A=0\quad ; W_{A\to B}(\vec{R})=0\Rightarrow \frac{1}{2}mv_B^2=W_{A\to B}(\vec{P})$$

$$W_{A\to B}(\vec{P})=mgh_1=mgr(1-\cos\alpha)\Rightarrow \frac{1}{2}mv_B^2=mgr(1-\cos\alpha)$$

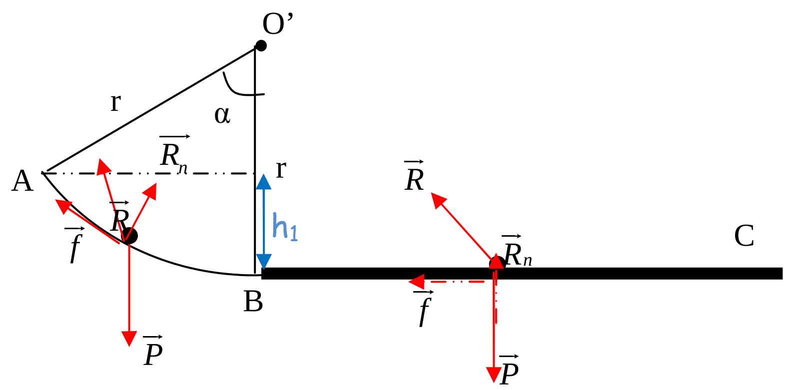
$$\Rightarrow \boxed{v_B=\sqrt{2gr(1-\cos\alpha)}}$$

$$v_B=\sqrt{2\times 9,8\times 5.10^{-2}(1-\cos 60)}\Rightarrow v_B=0,7m.s^{-1}$$

بين B و C ، $\vec{P}+\vec{R}=\vec{0}$ ،إذن حسب مبدأ القصور ،فإن الحركة مستقيمية منتظمة أي السرعة ثابتة : $v_C=v_B\Rightarrow v_C=0,7m.s^{-1}$

.2

.2.1



$$E_{CB}-E_{CA}=\sum W_{A\to B}(\vec{F})\Rightarrow \frac{1}{2}mv_B^2-\frac{1}{2}mv_A^2=W_{A\to B}(\vec{P})+W_{A\to B}(\vec{R})$$

$$v_A=0$$

$$W_{A\to B}(\vec{P})=mgr(1-\cos\alpha)$$

$$W_{A\to B}(\vec{R})=W_{A\to B}(\vec{R}_N)+W_{A\to B}(\vec{f})$$

$$W_{A\to B}(\vec{R}_N)=0$$

$$W_{A\to B}(\vec{f})=-f.\widehat{AB}\Rightarrow W_{A\to B}(\vec{f})=-f.r.\alpha$$

$$\Rightarrow \frac{1}{2}mv_B^2=mgr(1-\cos\alpha)-f.r.\alpha\Rightarrow \boxed{v_B=\sqrt{2\left(gr(1-\cos\alpha)-\frac{f.r.\alpha}{m}\right)}}$$

.2.2

$$E_{CC}-E_{CB}=\sum W_{A\to B}(\vec{F})\Rightarrow \frac{1}{2}mv_C^2-\frac{1}{2}mv_B^2=W_{B\to C}(\vec{P})+W_{B\to C}(\vec{R})$$

$$W_{B\to C}(\vec{P})=0$$

$$W_{B\to C}(\vec{R})=W_{B\to C}(\vec{R}_n)+W_{B\to C}(\vec{f})$$

$$W_{B\to C}(\vec{R}_n)=0\Rightarrow W_{B\to C}(\vec{f})=-f.BC\Rightarrow W_{B\to C}(\vec{R})=-f.BC$$

$$\Rightarrow \frac{1}{2}mv_C^2-\frac{1}{2}mv_B^2=-5f.r\Rightarrow v_C=\sqrt{v_B^2-\frac{10f.r}{m}}$$

$$\Rightarrow v_C=\sqrt{2\left(gr(1-\cos\alpha)-\frac{f.r.\alpha}{m}\right)-\frac{10f.r}{m}}\Rightarrow \boxed{v_C=\sqrt{2\left(gr(1-\cos\alpha)-\frac{f.r}{m}(\alpha+5)\right)}}$$

.2.3

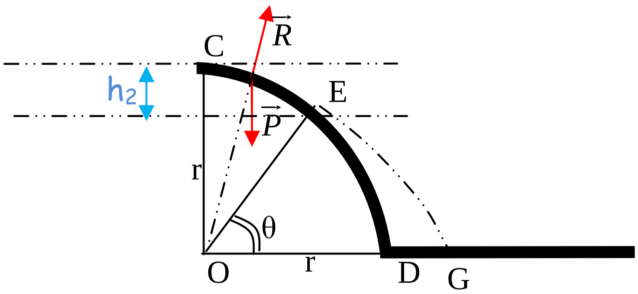
$$v_C=0\Rightarrow gr(1-\cos\alpha)-\frac{f.r}{m}(\alpha+5)=0\Rightarrow \boxed{f=\frac{mg(1-\cos\alpha)}{(\alpha+5)}}$$

$$f=\frac{80\times 9,8(1-\cos 60)}{\left(60\times\frac{\pi}{180}+5\right)}\Rightarrow \boxed{f=64,8N}$$

.3

.3.1

$$E_{CE}-E_{CC}=\sum W_{A\to B}(\vec{F})\Rightarrow \frac{1}{2}mv_E^2-\frac{1}{2}mv_C^2=W_{C\to E}(\vec{P})+W_{C\to E}(\vec{R})$$



$$W_{C\to E}(\vec{P})=mgh_2$$

$$h_2=r-r\sin\theta=r(1-\sin\theta)\Rightarrow W_{C\to E}(\vec{P})=mgr(1-\sin\theta)$$

$$W_{C\to E}(\vec{R})=0\Rightarrow \frac{1}{2}mv_E^2=mgr(1-\sin\theta)\Rightarrow v_E=\sqrt{2gr(1-\sin\theta)}$$

.3.2

$$v_E=\sqrt{2gr(1-\sin\theta)}\Rightarrow 2gr(1-\sin\theta)=v_E^2\Rightarrow 1-\sin\theta=\frac{v_E^2}{2gr}\Rightarrow \boxed{\sin\theta=1-\frac{v_E^2}{2gr}}$$

$$\sin\theta=1-\frac{0,57^2}{2\times 9,8\times 5.10^{-2}}=0,66\Rightarrow \theta\simeq 42°$$

.4 $E_{CG} - E_{CE} = \sum W_{A\to B}(\vec{F}) \Rightarrow \frac{1}{2}mv_G^2 - \frac{1}{2}mv_E^2 = W_{A\to B}(\vec{P})$

$W_{C\to G}(\vec{P}) = mgr\sin\theta \Rightarrow \frac{1}{2}mv_G^2 - \frac{1}{2}mv_E^2 = mgr\sin\theta \Rightarrow v_G^2 - v_E^2 = 2gr\sin\theta$

$\Rightarrow \boxed{v_G = \sqrt{v_E^2 + 2gr\sin\theta}}$

تطبيق عددي : $v_G = \sqrt{0,57^2 + 2\times 9,8\times 5.10^{-2}\times \sin 42} \Rightarrow v_G \simeq 1m.s^{-1}$